INVESTIGATING THE TIME PERIOD OF OSCILLATION OF A BIFILAR PENDULUM

Research Question (RQ) – How does the width of a symmetrical bifilar pendulum (a/ 0.040 m, 0.060 m, 0.080 m, 0.100 m, 0.120 m & 0.140 m) affect its time period (s) of oscillation when measured using a stopwatch (s/ ± 0.01 s)?

Introduction – As a prospective engineering student, I found the removal of the optional topics (Paper 3), especially option B engineering physics, for the current year's batch quite upsetting and disturbing. Therefore I took this IA as an opportunity to explore the concepts that are the backbone of engineering physics and pursue my future interests. After doing a summer course on Aerospace Engineering and learning about the utilization of mass moment of inertia in aircraft, I was introduced to the concept of bifilar pendulums and I personally found this system very intriguing especially because this simple mechanism has very useful practical applications in calculating the mass moment of inertia of irregular and regular objects. Moment of Inertia for an object is described as the tendency of an object to resist angular acceleration from torque around a particular axis. Moment of inertia has vital applications in aviation, for instance in flights, the contact surfaces of an aircraft are capable of producing aerodynamic forces and the moment of inertia helps evaluate the dynamic, stress and lastly the flutter conditions to which an aircraft is subjected to (*Habeck*). Drawn by my passion for aviation, I was interested in knowing how the mass moment of inertia can be calculated. Experimentally, the determination of the mass moment of inertia of an aircraft can be simplified by utilizing asymmetrical bifilar (rod), see below.

***Figure 1** – Diagrammatic view of a bifilar pendulum suspension ("Calculating the Moment of Inertia in Bifilar Pendulums.")*

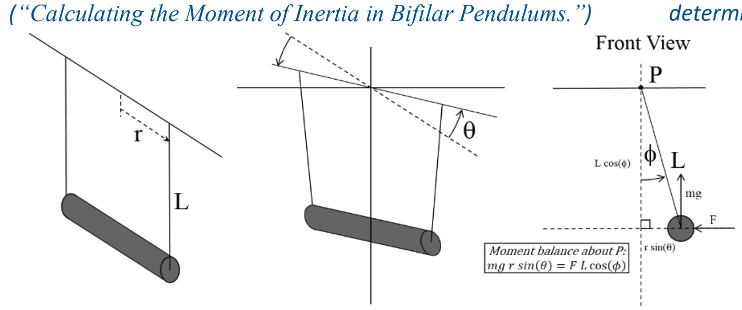


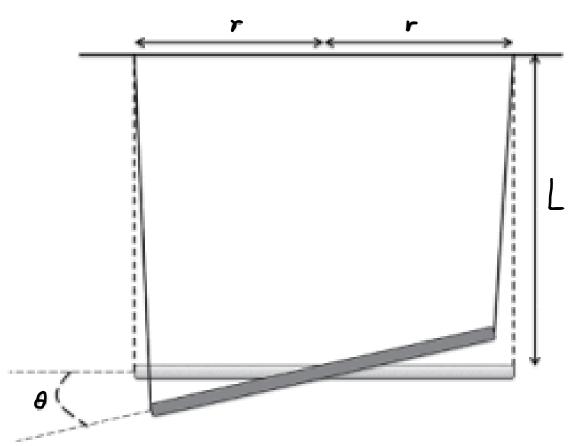
***Figure 2** – Real aircraft suspended like a bifilar pendulum for determining its mass moment of inertia (IBK Innovation GmbH & Co)*

Further research into this pendulum drew my curiosity into the factors which affect the time period of oscillation and as a result allowed me to pose the research question of investigating: how the thickness of the pendulum can affect the time period of its oscillation.

Background Research – Table 1.1 – Nomenclature

Symbol	Variable & S.I. Unit
τ	Net Restoring Torque ($N \cdot m$)
ϴ	Angle displaced in the x-direction ($radians$)
Φ	Angle displaced in the y-direction ($radians$)
L	Length of Suspension of the string (m)
I	Mass Moment of Inertia ($kg\ m^2$)
α	Angular Acceleration ($rad\ s^{-2}$)
a	Width of Bifilar Rod (m)
r	Radius of the Bifilar Rod (m)
b	Length/Diameter of the Bifilar Rod (m)
T	Time period of Oscillation (s)

***Figure 3** – Radius and Angle of Displacement of Bifilar Pendulum (Khan)*



A bifilar pendulum (in **figure 4**) is composed of a symmetrical rod (with uniform mass) suspended from two strings aligned parallel to each other (called "filars") which allow the pendulum to freely rotate about the central axis (in the $\hat{z}$ direction). The overall length of rod is equivalent to $2r$ and the length of the string/filar is L.

A horizontally twisted pendulum has no acceleration in the vertical plane ($\hat{z}$ direction); hence, the vertical forces must be balanced.

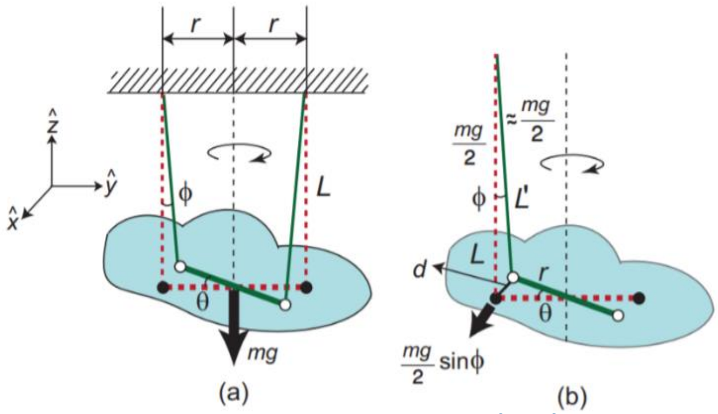


Figure 4 – Annotated Diagram of a Bifilar Pendulum (*Shaheen and Anwar 3*)

Theoretical Determination: *Deriving the time period of oscillation for a bifilar pendulum*

[Equation 1] Since there are no unbalanced forced in the vertical plane, the tension (F_T) in the string is equivalent to **$F_T = (\frac{mg}{2})$ Cos Φ** which is approximated to $\frac{mg}{2}$ assuming that the mass of the string is negligible/massless and that Φ is a small angle (indicating a displacement in the y-direction) where Cos Φ equals 1 ("Calculating the Moment of Inertia in Bifilar Pendulums.").

[Equation 2] Using the principle of geometry, it is evident that **L sin Φ = r sin Θ**. This equation can be rearranged in terms of Φ which is displacement in the y-direction.

$$\text{Sin } \Phi = \frac{r \sin\theta}{L} \approx \frac{r}{L}\theta \quad \text{because } \theta \text{ is approximated to be a small angle}$$

[Equation 3] Once the bifilar pendulum is horizontally twisted, it approaches its position of equilibrium because of the restoring torque that results from the horizontal component of the force of tension from the string ($\boldsymbol{F_T = -\frac{mg}{2}\ Sin\ \Phi}$). And since torque ($\tau = Fr$) is a product of the displacement from center of mass and perpendicular force, the overall restoring torque on the bifilar pendulum is equivalent to twice the product of r (moment arm) and the tension of the string *(Shaheen)*.

The factor of 2 is placed as a coefficient before the product (Fr) because restoring torque is experienced by both ends of the pendulum (with the same force and direction) which additively produce the oscillation of the bifilar pendulum and the overall torque.

$$\sum\tau = 2Fr = 2 \times (-\frac{mg}{2}\sin\Phi)r = -\text{mgr Sin } \Phi \text{ (shown in figure 3.b)}$$

This can be rewritten in terms of θ by substituting **equation 2**:

$$\sum\tau = -\text{mgr Sin } \Phi = -\text{mgr } (\frac{r}{L}\theta) = -\text{mgr}^2\frac{\theta}{L}$$

[Equation 4] According to **Newton's Second Law of Angular Motion**, the sum of torques is equivalent to the product of its mass moment of inertia and the object's angular acceleration.

$$\sum\tau = Ia = I(\frac{d^2\theta}{dt^2})$$

$\boldsymbol{a = \frac{d^2\theta}{dt^2}}$ because angular acceleration is the second derivative of angular displacement.

[Equation 5] Equating **equation 3 and 4** will yield *(Koken):*

$$Ia = I(\frac{d^2\theta}{dt^2}) = -mgr^2\,\frac{\theta}{L} \qquad \text{therefore } a = \frac{d^2\theta}{dt^2} = -(\frac{mgr^2}{I\,L})\theta$$

[Equation 6] According to the **equation of simple harmonic motion**, the acceleration is equivalent to $a = -\omega^2 x$, where ω is the angular velocity. Therefore $\omega^2 = \frac{mgr^2}{I\,L}$, assuming that x approximates to θ because of the small angle approximation *(Shaheen)*.

Rearranging this equation will yield $\omega = \sqrt{\frac{mgr^2}{I\,L}}$.

[Equation 7] Since time period (T) = $\frac{2\pi}{\omega}$. Therefore T = $\frac{2\pi}{\sqrt{\frac{mgr^2}{I\,L}}} = 2\pi\sqrt{\frac{I\,L}{mgr^2}} = \frac{2\pi}{r}\sqrt{\frac{I\,L}{mg}}$

[Equation 8] The mass moment of inertia for rectangular shaped rod (with uniform mass) is given by

$$I = \frac{m(a^2+b^2)}{12}$$

[Equation 9] Substituting **equation 8** for I into **equation 7** will yield *(Shaheen)*:

$$T = \frac{2\pi}{r}\sqrt{\frac{\left[\frac{m(a^2+b^2)}{12}\right]L}{mg}} = \frac{2\pi}{r}\sqrt{\frac{(a^2+b^2)\,L}{12g}} = \frac{2\pi}{2r}\sqrt{\frac{(a^2+b^2)\,L}{3g}} = \frac{\pi}{r}\sqrt{\frac{(a^2+b^2)\,L}{3g}}$$

∴ The time period (s) of oscillation for a bifilar pendulum is equivalent to $\frac{\pi}{r}\sqrt{\frac{(a^2+b^2)\,L}{3g}}$

Hypothesis – Since the time period of oscillation for bifilar pendulum is T = $\frac{\pi}{r}\sqrt{\frac{(a^2+b^2)\,L}{3g}}$ [Derived above].

If the thickness/width of the pendulum is changed then bifilar pendulum with the greatest width will have the greatest time period for completing one oscillation when the environment the experiment is conducted in, the type and length of string and the radius of the pendulum are kept constant because:

$$T^2 = \left[\frac{\pi}{r}\sqrt{\frac{(a^2+b^2)\,L}{3g}}\right]^2 = \frac{(a^2+b^2)\,\pi^2 L}{9gr^2} = \left(\frac{\pi^2 L}{9gr^2}\right)a^2 + \frac{b^2\,\pi^2 L}{9gr^2}$$

y = m x + c (Equation for a linear relationship)

Therefore, there is a positive correlation of T^2 (time period) with a^2 (thickness/width of the pendulum). Hence when the square of the width of the pendulum will increase the time period squared linearly. Thus, a greater width will yield a greater time period for one oscillation and vice versa.

Independent Variable (IV) – The width of the bifilar pendulum (m/± 0.001 m) which could be manipulated by using multiple rods of (2 cm diameter each) and sticking them together in one plane to vary the thickness. The length (0.300 m) and height of the rod (0.002 m) will remain constant. Range of IVs used: 0.020 m, 0.040 m, 0.060 m, 0.080 m, 0.100 m, 0.120 m & 0.140 m.

Dependent Variable (DV) – The time period (T) of an oscillation for the bifilar pendulum measured using a stopwatch (s) ± 0.01s. This will be calculated by recording the time taken for 15 oscillations and dividing it by 15, to determine the time period of one oscillation to minimize the random error.

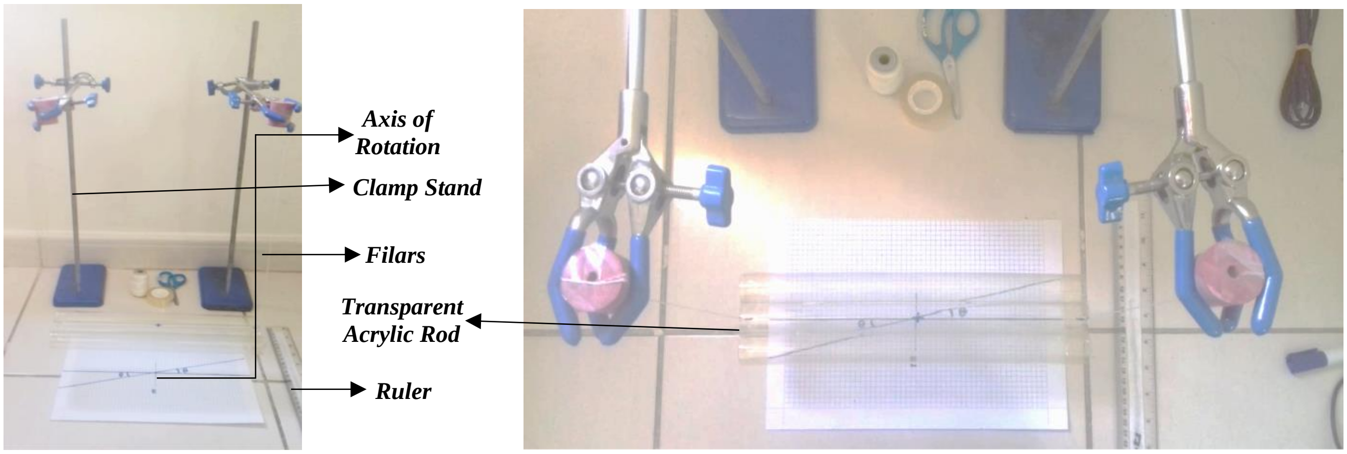
Controlled Variable (CVs) – Table 1.2 – The following variables will be controlled to ensure a fair test:

Controlled Variable	Why it is being controlled?	What happens if it was not controlled?	How can it be controlled?
Environment conditions	Environmental conditions such as whether, air resistance and wind can affect the data obtain by assisting the swing of the bifilar pendulum which will cause the time period of oscillation to be imprecise.	If the environment were changed and the air resistance and wind was varied than it can affect the time period of oscillation by assisting the motion of pendulum which will cause the results obtained to vary, increasing the random error on the graph.	This is controlled by conducting the experiment in a controlled environment such as a school laboratory for the entire investigation.
Type/Brand of String	The mass of the string is considered to be negligible, hence the use of a thicker string will cause the mass to increase which will yield inaccurate results.	If the type of string is not controlled than the tension experienced by the string and its mass will cause the overall torque to differ and hence the results obtained to vary.	This can be controlled by utilizing a thin string which is from the same string role and the same brand/type.
Radius of Pendulum	The time period of oscillation is proportional to the inverse of the radius of the bifilar; therefore, if the radius changes, it will cause the time to vary.	If a pendulum with a larger radius is used than the time period for one oscillation will reduce and vice versa, therefore if the radius varies, the results and the trend will be inaccurate.	The radius of the bifilar pendulum will be kept constant to 0.150 m and an overall length of 0.30 m using a ruler.
Length of Suspension	For a pendulum, the length of suspension is directly proportional to the square of the time period ($T^2 \propto L$).	Therefore, if the length of the string varies, it will cause the time period to vary with length and as result cause a high random error.	This will be controlled by utilizing a constant length of suspension of 0.30 m on both sides.
Shape of the Object Suspended	Different shaped objects may not have uniform mass and may have different moments of inertia calculations which will affect the time period.	If the overall moment of inertia increases about the central axis than the time period of oscillation would be greater and vice versa causing the time period to vary.	The same symmetrical bifilar rod will be utilized in all trials with a uniform mass.
Angle Displaced	The angle of displacement is assumed to be a relatively small angle (background research), for the small angle approximation to hold true.	If this angle is greater than the overall calculation will deviate from the actual value causing a discrepancy in the data.	The angle of displacement will be kept constant to an angle of 20°.

Apparatus – Table 1.3 – The following apparatus will be required for the investigation:

Equipment	Specification	Qty.	Uncertainty	Equipment	Specification	Qty.	Uncertainty
Clamp stand	With holder	x2	NA	Measuring Balance	-	x1	± 0.00001 kg
String	Full Role	x1	NA	Scissors	-	x1	NA
Ruler	30 cm length	x1	±0.001 m	Stopwatch	-	x1	± 0.01 s
Acrylic Tube	2cm diameter	x9	NA	Tape	Full Role	x1	NA

Experimental Setup – Figure 4 – A Diagram showing the experimental set-up of the apparatus.



4(a) *Annotated front-view of the set-up (Candidate, 2021)*

4(b) *Top view – aligning the center of the rod with the equilibrium position on the chart (Candidate, 2021)*

Safety/Risk Assessment – There are no significant risks involved within this experiment, however the following should be taken into consideration before commencing:

- Clamp stands are heavy, should be kept away from edges because if they fall from a height, they can cause minor injuries to the foot.
- Scissors are sharp and pointy, so handle with care and avoid pointing at someone.
- **Environmental concern** – Used tape and plastic acrylic tubes should be disposed correctly.

Concerns and Uncertainty Considerations - Otherwise, this experiment is safe and hence environmental and ethical concerns that are involved within this experiment are minimal. The impact of uncertainties is taken into consideration but utilizing dimensions of rod that would produce relatively small percentage errors and minimize the impact of uncertainties on the reading.

Preliminary Measurements – Table 1.4 – The table below shows the preliminary measurements collected before conducting the actual investigation along with their uncertainties:

Radius of the Bifilar (r)	*Length of Suspension (L)*	*Mass of the Rod (m)*	*Diameter of the rod (b)*
0.150 m ± 0.001 m	0.300 m ± 0.001 m	0.0132 kg ± 0.00001 kg	0.020 m ± 0.001 m

Methodology –

1. Place the both the clamp stands on flat surface 30 cm apart and adjust the holder's height to a perpendicular distance of 35 cm to the ground measured using a ruler.
2. Take a 0.3 m length acrylic tube (with 0.020 m diameter) and pass a string through it.
3. Pass the string through the holes in the rubber bung and tie a knot. Ensure that the suspended length of the string is 0.3 m on each side of between the bung and the tube.
4. Clamp the rubber bung to the clamp stand.
5. Using a permanent marker, highlight the center of the tube (which is 0.15 m from each side). This is the center of rotation axes (this needs to be controlled for each displacement).
6. Take a scratch paper and place it underneath the pendulum; mark the resting equilibrium position of the bifilar pendulum and label a small angle ϴ = 20° with a line using a protractor.

7. Displace the pendulum by the angle ϴ indicated on the paper (to control the angular displacement) and start the stopwatch as soon as the pendulum is released.
8. Record the time for 15 oscillations in an appropriate table.
9. Repeat steps 2-8 to conduct 5 trials.
10. Calculate the average for all the trials of 15 oscillations followed by the average for the time period for one oscillation.
11. Repeat steps 2-10 for all other ranges of IVs by taping the tubes together to increase their width by a constant interval (0.040 m, 0.060 m, 0.080 m, 0.100 m, 0.120 m & 0.140 m).

Justification of Uncertainties: The uncertainty for each measurement is calculated from the following:

- **Meter ruler –** The meter ruler used has a smallest division of 1 mm (which is equivalent to 0.001 m). The uncertainty for any analogue equipment (such as a ruler) is equivalent to half of the smallest division meaning the uncertainty is ± 0.0005 m for the ruler. Since the measurement is taken from initial starting zero with an uncertainty of ± 0.0005 m to the ending measurement with an uncertainty of ± 0.0005 m. The total absolute uncertainty for any measurement on a ruler is ± 0.001 m.
- **Digital Equipment –** The uncertainty for any digital equipment is equivalent to the last significant figure in the measurement of a digital apparatus.
    - Therefore, the absolute uncertainty in the stopwatch is ± 0.01 s.
    - And the absolute uncertainty in the electronic measuring scale is ± 0.00001 kg.

Qualitative Data – Following observations were made whilst doing the investigation:

- While doing the experiment, I realized that it was difficult to allow the tubes to remain in one single plane horizontal to the ground because of the weight. So, I had to tape the string to ensure that the plane of all the tubes is parallel to the ground instead of being vertical.
- In addition, I noticed while taping the acrylic tubes together, there was a slight non-uniform distribution of mass because adding unequal amount of tape to both sides of the pendulum. This can have impact on the mass moment of inertia and can cause slight deviations because the formula derived (in background research) assumes that mass is uniformly distributed. Unsymmetrical distribution of mass can also cause the tension on the individual strings to vary which can have slight effects on the time period of oscillation.
- There was some refraction and parallax error when displacing the bifilar pendulum with angle of ϴ because of the light refracting on the line labeled on cardboard paper, causing the angle of displacement of the bifilar pendulum to fluctuate between trials.

Raw Data – Table 1.5 – How does the width of a symmetrical bifilar pendulum (m/ 0.040 m, 0.060 m, 0.080 m, 0.100 m, 0.120 m & 0.140 m) affect its time period for completing 15 oscillations when measured using a stopwatch (s/ ± 0.01 s)?

Width of the Bifilar Pendulum a/m (Δa = ± 0.001 m)	**Time taken (s) for completing 15 oscillations** t/s (Δt = ± 0.01 s)				
	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**
0.020	0.53	0.62	0.47	0.51	0.49
0.040	0.96	0.98	0.84	0.82	0.95
0.060	1.04	1.02	0.92	0.95	0.89
0.080	1.42	1.34	1.47	1.38	1.36
0.100	1.52	1.63	1.58	1.49	1.61
0.120	2.10	2.08	2.14	2.22	2.16
0.140	2.27	2.18	2.26	2.16	2.22

Sample Calculations – Table 1.6 – Note all sample calculation have been shown for the IV of 0.020 m only. The same calculations will be repeated for all other ranges of IV using their respective data.

Calculation	Method of Calculation	Sample Calculation – (shown IV 1)
Propagating absolute uncertainties for the width of the pendulum squared	Calculate the percentage uncertainty of *a* $\Delta\ a = \frac{Absolute\ Uncertainty}{Actual\ Reading} \times 100$ *Calculate percentage uncertainty in a²* $\Delta\ a^2 = 2 \times (\Delta a)$ Absolute Uncertainty in width squared (a^2) $\Delta\ a^2 = \frac{\%age\ in\ \Delta a^2}{100} \times a^2$	$\%age\ \Delta a = \frac{0.001}{0.020} \times 100 = 5\%$ $\%age\ \Delta a^2 = 2 \times (5\%) = 10\ \%$ Absolute Uncertainty in a^2 $\mathbf{\Delta a^2} = \frac{10}{100} \times (0.02)^2 = 0.00004$
Average time for 15 oscillations	$Average = \frac{\sum of\ all\ the\ trials}{No.\ of\ trials} = \frac{T1+T2+T3+T4+T5}{5}$	$Avg = \frac{0.53+0.62+0.47+0.51+0.49}{5} = 0.52\ s$
Average time for 1 oscillation	$T = \frac{Average\ Time\ for\ 15\ Oscillations}{15}$	$Average\ Time\ Period = \frac{0.52}{15} = 0.0347\ s$
Average Time Period Squared	T^2 = (Average Time Period for 1 Oscillation)^2	$T^2 = 0.0347\ s \times 0.0347\ s = 0.001207\ s^2$
Uncertainty in Time Period squared (for error bars)	Uncertainty in Average time period for 1 oscillation $\Delta T = \left[\frac{Max-Min}{2}\right] \div 15$ Uncertainty in time period *squared* for 1 oscillation is equivalent to: $\Delta T^2 = 2 \times \left[\frac{\Delta T}{T}\right] \times T^2 = 2 \times \Delta T \times T$	$\Delta T = \left[\frac{0.62-0.47}{2}\right] \div 15 = 0.005\ s$ $\Delta T^2 = 2 \times 0.005 \times 0.0347 = 0.001048$ $T^2 = 0.001207 \pm 0.001048\ s^2$

Processed Data – Since the derived equation is $T^2 = \left(\frac{\pi^2 L}{9gr^2}\right) a^2 + \frac{b^2\ \pi^2 L}{9gr^2}$. Time period for one oscillation and the width of the pendulum must be squared to produce a linear relationship according to derived formula. Hence the data is processed to present the square relationship of T^2 and a^2 with their relevant uncertainties.

Table 1.7 - How does the width of of a bifilar pendulum (m/ 0.040 m, 0.060 m, 0.080 m, 0.100 m, 0.120 m & 0.140 m) affect its average time period (s) of oscillation when measured using a stopwatch (s/ ± 0.01 s)?

Width of the Bifilar Pendulum a/m ($\Delta a = \pm 0.001$ m)	**Width of Bifilar Pendulum squared** a^2/m^2	**Absolute Uncertainty in width squared** (Δa^2)	**Average time (s) for 15 Oscillations** t/s ($\Delta t = \pm 0.01$ s)	**Average Time Period for 1 Oscillation** T/s ($\Delta T = \pm 0.01$ s)	**Average time period squared** T^2/s^2	**Uncertainty in time period*** (ΔT^2)
0.020	0.00040	± 0.00004	0.52	0.0347	0.001207	± 0.001048
0.040	0.00160	± 0.00008	0.91	0.0607	0.003683	± 0.001941
0.060	0.00360	± 0.00012	0.96	0.0643	0.004143	± 0.001928
0.080	0.00640	± 0.00016	1.39	0.0927	0.008587	± 0.002416
0.100	0.01000	± 0.00020	1.57	0.1049	0.011015	± 0.002923
0.120	0.01440	± 0.00024	2.14	0.1429	0.020427	± 0.003995
0.140	0.01960	± 0.00028	2.22	0.1480	0.021923	± 0.003253

***Note –** The shaded columns (**on Table 1.7 on the previous page**) will be graphed on logger pro to produce a *linear relationship* with their respective absolute uncertainties and associated error bars. Large number of decimal places are shown in the table because if it is rounded, it will be too difficult and small to be observed on the graph.

Graph – A scatter graph has been used to represent the results obtained because it is a numerical collection of data where the independent and dependent variable are both quantitative (numerical) which is best represented by a scatter graph to draw a line of best fit and illustrate the trend between the width and time period of oscillation.

Justification of Error Bars used on the Graph:

- **Horizontal Error Bars:** Propagated uncertainty in the width squared by using the absolute uncertainty of the ruler used for measuring the width of the bifilar pendulum.
- **Vertical Error Bars:** Equipment (stopwatch) uncertainty was too small to be shown so experimental uncertainty is used to draw error bars. Experimental uncertainty from the values obtained ($\frac{max-min}{2}$) which was further propagated to produce uncertainty for time period squared (see sample calculations).

The graph illustrates the following relation $T^2 = \left(\frac{\pi^2 L}{9gr^2}\right) a^2 + \frac{b^2\,\pi^2 L}{9gr^2}$ determined experimentally.

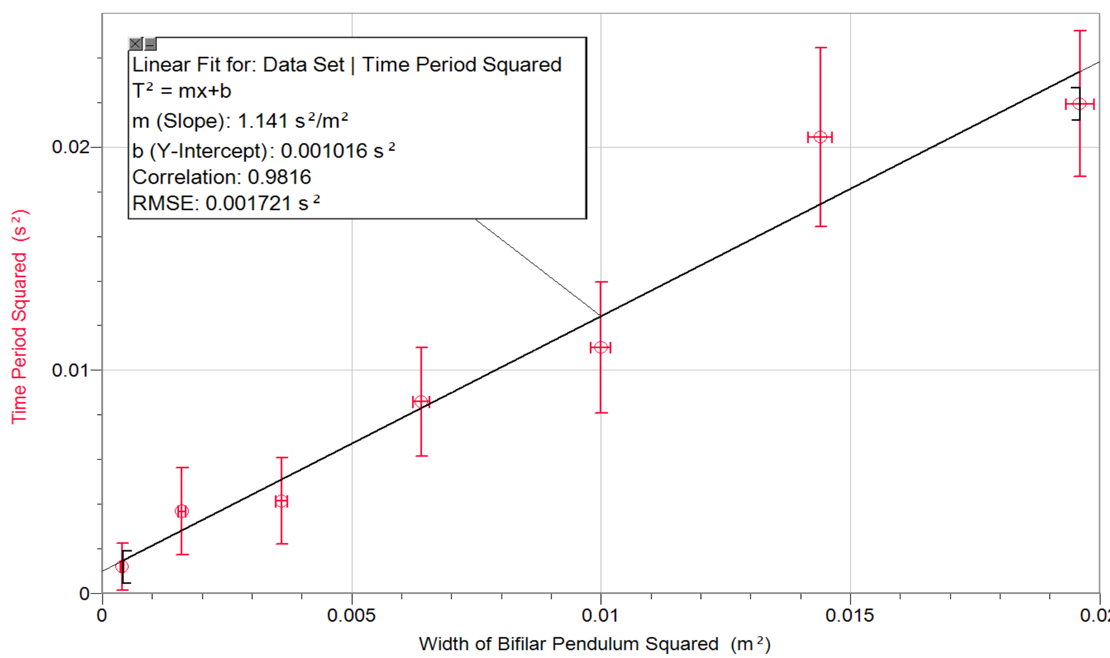


Figure 5 – Graph of the time Period of Oscillation Squared Vs. Width of Bifilar Pendulum Squared made on Logger Pro (Candidate, 2021)

*The horizontal error bars too small to be clearly visible for the first two range of IVs (width squared).

According to the graph, there is a positive linear correlation between the time period of oscillation squared and the width of the pendulum squared. This is evident because the line of best fit is a straight line passing through all error bars. The fact that line of best fit crosses all error bars increases the precision of the results obtained and suggests that there was small random error. The graph shows evidence of systematic errors because there is shift in the theoretical y-intercept and gradient in comparison to the actual gradient and intercept. Furthermore the coefficient of determination (r^2) is extremely high being 0.9816 (close to 1) which means that there is a strong linear association between the two variables (i.e. the width square and time period squared).

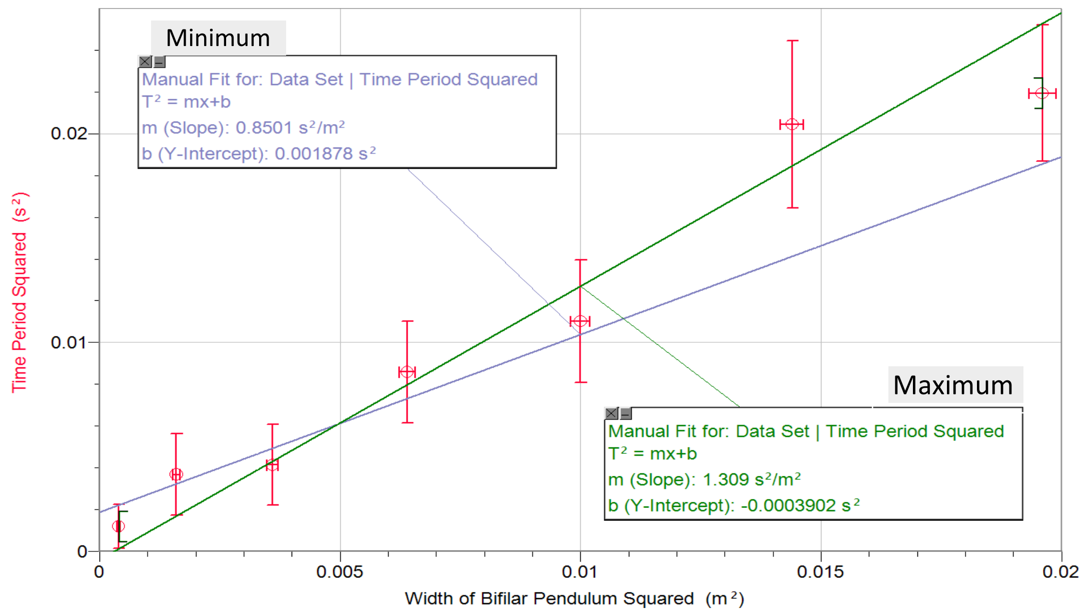


Figure 6 – Maximum and Minimum Lines Drawn using the error bars on Logger Pro (Candidate, 2021).

The equations of these line will determine the uncertainty in the gradient and y-intercept which will be discussed in the data analysis. All error bars intersect with the maximum and minimum lines which denotes that the results obtained are significantly precise.

Data Analysis – The graph (refer to **figure 5**) shows the data obtained from experimentally investigating the time period of oscillation after varying the width of the bifilar pendulum. According to the graphs produced from data obtained it is clear that there exists a positive linear correlation (as predicted within the hypothesis) between the time period and width of the pendulum both squared. The values obtained can be compared with the theoretical values from the derived formula to evaluate the accuracy of the practical. The theoretical formula for time period is $T^2 = \left(\frac{\pi^2 L}{9gr^2}\right) a^2 + \frac{b^2\,\pi^2 L}{9gr^2}$.

y = m x + c (General Equation for a linear relationship)

- Analyzing the Gradient of the Graph
  - Theoretical Gradient = $\frac{\pi^2 L}{9gr^2} = \frac{0.3\pi^2}{9 \times 9.81 \times 0.150^2}$ = **1.49 $s^2\,m^{-2}$**
  - Actual Gradient (from the Logger Pro Graph: **Figure 5**) = **1.14 $s^2\,m^{-2}$**
  - Absolute Uncertainty in the gradient (using the gradient of max/min lines in **Figure 6)**

    $\frac{m_{max} - m_{min}}{2} = \frac{(1.309)-(0.8501)}{2} = 0.229745 \quad \approx \pm 0.23\ (2\ s.f.)$

    $Percentage\ Uncertainty = \frac{0.23}{1.14} \times 100 = 20\%$

    Gradient = $1.14 \pm 0.23\ s^2\,m^{-2} = 1.14 \pm 20\,\%\ s^2\,m^{-2}$
  - Percentage Error/Difference in the Gradient

$$\left|\frac{Theoretical\,Value - Experimental\,Value}{Theoretical\,Value} * 100\right| = \left|\frac{1.49-1.14}{1.49} * 100\right| = 23.5\ \%$$

Using the absolute uncertainties of the gradient, it is clear that the experimental range of the gradient is between 0.91 $s^2\,m^{-2}$ to 1.37 $s^2\,m^{-2}$ whilst the theoretical value for the gradient is 1.49 $s^2\,m^{-2}$. This suggests that the theoretical gradient doesn't fit within the range of experimental gradient.

Since the percentage uncertainty of the gradient (20 %) is less than the percentage error/difference **(23.5 %),** it decreases the validity of the results and methodology. It suggests that the random errors cannot justify the difference between the actual gradient and the theoretical gradient, hence is a result of the systematic errors. This can be due to assumptions made while deriving the formula of the width vs. the time period, firstly the angle of displacement in the vertical plane was assumed to be very small, the string was assumed to massless, and the rod was assumed to have symmetrical/uniform distribution throughout the bifilar.

- Analyzing the Y-Intercept
  - Theoretical Y-Intercept = $\frac{b^2\,\pi^2 L}{9gr^2} = \frac{0.02^2\times\pi^2\times 0.3}{9\times 9.81\times 0.150^2}$ = **5.96 × 10^{-4} s^2**
  - Actual Y-Intercept (from the Logger Pro Graph: **Figure 5**) = $\mathbf{10.16 \times 10^{-4}}$ **s^2**
  - Uncertainty in the Y-Intercept (from max/min lines in **Figure 6**)

$$\frac{c_{max} - c_{min}}{2} = \frac{(0.001878) - (-0.0003902)}{2} = 0.0011341 \approx \pm\, 0.0012\ s^2\ (2\ s.f.)$$

  - **Percentage Error** in the Y-Intercept

$$\left|\frac{Theoretical\ Value - Experimental\ Value}{Theoretical\ Value} * 100\right| = \left|\frac{(5.96\times 10^{-4}) - (10.16\times 10^{-4})}{(5.96\times 10^{-4})} * 100\right| = 70.47\ \%$$

Conclusion – The trend obtained with in the graph shows a positive linear correlation and relationship between the time period squared and the width squared of the bifilar pendulum as expected and supports the hypothesis. The line of best fit within the graph passes through all error bars, with a coefficient of determination (r^2) of 0.99724 (a value which is extremely close to 1), suggesting that the strength of the association between the two variables is extremely strong thus supporting the linear relationship between the time period of oscillation and width of the bifilar pendulum. All error bars fall within the maximum and minimum lines except one at the width squared of 0.014, which could have been due to the fact that the angle of displacement kept fluctuating because of the parallax error hence yielding a greater time period squared. Furthermore it was difficult to maintain the center axis of rotation for rods with such large diameters because of the uneven distribution of weight across the bifilar rod. This is the reason why there is a huge percentage error (**70.5 %**) in the y-intercept. Furthermore, according to the small angle approximation, the Maclaurin series for $Sin\ \theta$:

$$Sin\ \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Hence at small angles, it is sensible to assume that $Sin\ \theta = \theta$, but at larger angles the values start to deviate (because of the second term of the series) which could have caused fluctuations in the time period of oscillation. Thus, the large percentage error in the gradient of the graph of **23.5 %** is attributable to the fact that the angle was not small enough in such a scaled experiment. A moderately scaled experiment was chosen to ensure that the absolute percentage uncertainties for variables are low but the displacement approximates to θ. The scale of the experiment is appreciated because it yielded relative low percentage uncertainties (of 20%). Examining the graph, there is evidence of a large systematic error that is caused by the numerous assumptions made during the derivation of the equation, such as the small angle approximation, the human reaction time, and the uniform distribution of mass within the rod. The results obtained are relatively precise (due to lower percentage uncertainty/random error) but less valid because the percentage error exceeds the percentage uncertainty **(23.5 % >> 20%).** Placing this in a scientific context, this investigation certainly supports the derivation that time periods squared is linearly correlated with the width of bifilar pendulum squared, and therefore the width can certainly affect the calculations of the mass moment of inertia.

Strengths – The following strengths are identified within this investigation:

- **Sufficient Repetition of trials & Range of IVs –** Since a sufficient number of trials were conducted for each range of IV, it allowed for a reliable trend to be obtained and allowed sufficient conclusions to be drawn since the impact of random error was minimized and any anomalies were easily identified and eliminated. The range of IVs was sufficient enough to produce a visible trend on the graph and allow conclusions to be drawn.

- **Precision of Results & Collection of relevant data–** The absolute and percentage uncertainty within the time period were smaller in comparison to percentage error which indicates the reliability of the results and increases the precision and reliability of the data allowing the line of best fit to pass through all the error bars. Since the graph supports the expected trend, it suggests that relevant data was collected.

Evaluation/Improvements – Table 1.8 – Limitations to this investigation and their improvements

Limitation	How it affects the data?	How it can be improved?
Error: Experimental Limited Range of IVs	7 range of independent variables were tested to measure the effect on the time period of oscillation. A greater range of IVs will produce a more reliable and visible trend which will increase the validity of data and allow better conclusions to be drawn.	This can be improved by increasing the range IV's to 10 with increments intervals of 0.002 m width.
Error: Random Number of Trials Conducted	Only 5 trials were conducted to measure the time period of oscillation for different widths of pendulums. 5 trials are sufficient for drawing conclusions, identifying, and eliminating anomalies and producing a sufficient trend, however a larger number of trials will increase the precision and reliability and reduce the random error.	This will be improved by an increase in the number of trials to 7-10 trials instead of just 5 trials which will increase the precision of results.
Error: Parallax Error While Displacing the Acrylic Tube	When displacing the acrylic tube, there was a cardboard paper marked underneath the tube to control the angle of displacement and to keep it constant. But due to the refraction of light between the two mediums and the curvature of the tubes, the angle of displacement varied with each trial which led to an inaccuracy of the results obtained as it causes a deviation within the derivation of the formula.	This can be improved by a using a rectangular rod with (no curvature) and using thin pillars attached to the ground to mark the angle of displacement relative to the equilibrium position.
***Error:* Human** Time Lapse between 5 oscillations and stopping the stopwatch	There is evidence of human error because there was a time lapse between during the duration when the 15 oscillations were actually complete and the when the stopwatch was stopped. This human error would have contributed to an increase in systematic error because the obtained value is always greater than real value that should have been recorded.	This can be improved by increasing the number of oscillations to about 20-25 which will ensure the impact of this human reaction time would be smaller.
***Error*: Random** Maintaining the center of rotation axis	Although a tube with a uniform mass was taken to ensure that center of rotation axis remains constant, due to adding tape for attaching multiple tubes, the mass was unevenly distributed causing the center of rotation axis to vary which could have yielded the time period to vary hence producing unreliable data.	This can be improved by equally spacing out the tubes & using equal lengths of tape on both sides to avoid uneven distribution.
***Error:* Experiment** Scale of the Experiment	This experiment was conducted on a relatively small scale to minimize the disposal of plastic equipment and thread, however if it were to be conducted on a larger scale then the impact of uncertainties of the time period would have been much lower thus increasing the precision of results.	This can be improved by using scaled tubes with a larger cross-sectional diameter, greater length of wire, larger bifilar radius.

Extensions – There are a few potential extensions for this investigation to understand the variables that affect the motion of a bifilar pendulum: For example, how the length of the string can affect the time period of oscillation. Alternatively, the motion of an unsymmetrical bifilar pendulum (with a non-uniform mass) can be examined and how it affects the mass moment of inertia. This can have more valuable applications to aeronautical engineering because the mass is not distributed evenly within an airplane. In addition, the motion of symmetrical bifilar pendulum with non-central axis of rotation (at any arbitrary point from the center of mass) can be investigated in future by applying the calculations of the parallel axis theorem.

Reflection: Ultimately, investigating the motion of a bifilar pendulum allowed me to explore engineering physics and rotational dynamics which helped me achieve my initial goal and future interests as a prospective mechanical engineer.

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